WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 3rd Semester Examination, 2022-23

MTMACOR07T-MATHEMATICS (CC7)

Time Allotted: 2 Hours | Full Marks: 40

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *four* questions from the rest

1. Answer any ***four*** questions from the following: 2×4 = 8

(a) If $y = 4x^6 - 5x$, find the percentage error in y at $x = 1$, if the error in $x = 0.04$.

(b) What are the advantages and disadvantages of the Bisection method for finding a root of the equation $f(x) = 0$.

(c) Write down Newton's forward interpolating polynomial with usual notations.

(d) For any positive number k, prove that $y_k = \sum_{i=0}^{k} \binom{k}{i} \Delta^i y_0$, Δ being the forward difference operator.

(e) Write down the formula for Weddle's rule for evaluating $\int_a^b f(x)dx$ using 12 subintervals. Is there any restriction on the number of subintervals for this particular rule?

(f) Given $\frac{dy}{dx} = x^3 + y$, $y(0) = 1$. Compute $y(0.02)$ by Euler's method, correct upto four decimal places taking step length 0.01.

(g) Write 'T' for True and 'F' for False statement.

(i) In Simpson's $\frac{1}{3}$rd rule $\int_{x_0}^{x_2} y dx = \frac{3h}{4}[y_0 + 4y_1 + y_2]$

(ii) $\left(\frac{dy}{dx}\right)_{x=x_0} = \frac{1}{h}\left[\Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0 - \cdots\right]$

2. (a) The percentage error in R, which is given by $R = \frac{r^2}{2h} + \frac{h}{2}$, is not allowed to exceed 0.2%. Find allowable error in r and h when $r = 4.5$ cm and $h = 5.5$ cm. 4+4

(b) Perform three iterations of the Newton-Raphson method to obtain the approximate value of $(17)^{1/3}$ starting with the initial approximation $x_0 = 2$.

3. (a) Find $\Delta^{10}[(1-ax)(1-bx^2)(1-cx^3)(1-dx^4)]$ 2+6

(b) Find the value of $f(x)$ for $x=2.55$ from the following data

x	1	2	3	5
$f(x)$	3	10	29	127

4. (a) Design an algorithm to compute the HCF and LCM of two numbers, provided by user. 4+4

(b) Evaluate the integral $\int_0^5 \frac{dx}{4x+5}$ by Weddle's Rule.

5. (a) Using LU decomposition method, find the inverse of the matrix 5+3

$$\begin{bmatrix} 2 & -2 & 4 \\ 2 & 3 & 2 \\ -1 & 1 & -1 \end{bmatrix}$$

(b) From the following table, find the area bounded by the curve and x-axis from $x=7.47$ to $x=7.52$ by trapezoidal rule:

x	7.47	7.48	7.49	7.50	7.51	7.52
$f(x)$	1.93	1.95	1.98	2.01	2.03	2.06

6. (a) Find the largest eigen-value and the corresponding eigenvector of the following matrix by power method (correct upto 2D) 5+3

$$\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

(b) Establish numerical differentiation formula based on Newton's forward difference formula for equispaced arguments.

7. (a) Solve the following system by Gauss Elimination method 4+4

$$x_1 + x_2 + 2x_3 = 4$$
$$x_1 + 2x_2 + 3x_3 = 6$$
$$2x_1 + 3x_2 + x_3 = 6$$

(b) Use method of successive approximation for finding approximate solution of the equation $\frac{dy}{dx} = x - y$, $y(0)=1$.

8. Describe the power method for finding the largest (in magnitude) eigen value of a real square matrix A. How can the least eigen value (in magnitude) be obtained by using power method? Explain it mathematically. 8

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